$SU(2N_f) \otimes O(3)$ light diquark symmetry and current-induced heavy baryon transition form factors

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Abstract

We study the current-induced bottom baryon to charm baryon transitions in the Heavy Quark Symmetry limit as $m_q \to \infty$. Our discussion involves s-wave to s-wave as well as s-wave to p-wave transitions. Using a constituent quark model picture for the light diquark system with an underlying $SU(2N_f) \otimes O(3)$ symmetry and the heavy quark symmetry we arrive at a number of new predictions for the reduced form factors that describe these transitions.

1. Introduction

The Heavy Quark Effective Theory (HQET) formulated in 1990[1] is so well known by now that it no longer needs an extensive introduction. The HQET provides a systematic expansion of QCD in terms of inverse powers of the heavy quark mass. The leading term in this expansion gives rise to a new spin and flavour symmetry at equal velocities, termed the Heavy Quark Symmetry. Corrections to the Heavy Quark Symmetry limit can be classified and evaluated order by order in $1/m_Q$ by considering the contributions of the nonleading terms in the effective HQET fields and the HQET Langrangian.

In this paper we will only be concerned with the leading order term in the HQET expansion, i.e. in the Heavy Quark Symmetry limit. In this limit the dynamics of the light and heavy constituents decouple and the calculation of the Heavy Quark Symmetry predictions for transition amplitudes essentially amounts to an angular momentum coupling exercise, however involved it may be. It is then not surprising that such calculations have been done even before the conceptual foundations of HQET had been laid down in 1990. For example, the Heavy Quark Symmetry structure of current-induced charm baryon transitions had been written down as early as 1976[2]. As concerns the angular momentum coupling calculations these can either be done using Clebsch-Gordan coefficents or, more compactly, using the Wigner 6-j-symbol calculus. Still a different method is to use covariant spin wave functions or the Bethe-Salpeter (B-S) formalism. In our previous works, Refs. [3]-[7], we have used this formalism to derive the consequences of the heavy quark symmetry for weak transitions of hadrons of arbitrary spin. This is the approach taken by most workers in the field and as well as in this paper.

At present most of the attention of experimentalists and theoreticians working in heavy quark physics is directed towards the application of HQET in the meson sector where data is starting to become quite abundant. This data will be supplemented in the not-too-distant future by corresponding data on heavy baryon decays and there will be a need to analyze this data in terms of HQET as applied to heavy baryons. In the present work we disuss the structure of flavour changing bottom baryon to charm baryon decays in the Heavy Quark Symmetry limit as $m_Q \to \infty$. We go beyond our previous works on the subject in that we treat the light diquark system in a constituent quark model approach with an underlying $SU(2N_f) \otimes O(3)$ symmetry. This results in a number of new predictions, which are consistent with but go

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beyond the Heavy Quark Symmetry predictions for the reduced form factors that describe current-induced bottom baryon to charm baryon transitions.

2. Classification of s- and p-Wave Heavy Baryon States

A heavy baryon is made up of a light diquark system (qq) and a heavy quark Q. The light diquark system has bosonic quantum numbers j^P with the total angular momentum $j = 0, 1, 2 \ldots$ and parity $P = \pm 1$. To each diquark system with spin-parity j^P there is a degenerate heavy baryon doublet with $J^P = (j \pm 1/2)^P$ ($j = 0$ is an exception). It is important to realize that the Heavy Quark Symmetry structure of the heavy baryon states is entirely determined by the spin-parity j^P of the light diquark system.

From our experience with light baryons and light mesons we know that one can get a reasonable description of the light particle spectrum in the constituent quark model picture. This is particularly true for the enumeration of states, their spins and their parities. As much as we know up to now, gluon degrees of freedom do not seem to contribute to the particle spectrum. It is thus quite natural to try the same constituent approach to enumerate the light diquark states, their spins and their parities. From the spin degrees of freedom of the two light quarks one obtains a spin 0 and a spin 1 state. The total orbital state of the diquark system is characterized by two angular degrees of freedom which we take to be the two independent relative momenta $k = \frac{1}{2}(p_1 - p_2)$ and $K = \frac{1}{2}(p_1 + p_2 - 2p_3)$ which can be formed from the two light quark momenta p_1 and p_2 and the heavy quark momentum p_3 [7]. The k-orbital momentum describes relative orbital excitations of the two quarks, and the K-orbital momentum describes orbital excitations of the center of mass of the two light quarks relative to the heavy quark. The details are in the next section. The (k, K) basis is quite convenient for two reasons. First, Copley, Isgur and Karl[8] have found that the (k, K) basis diagonalizes the Hamiltonian, when harmonic interquark forces are used. Second, it allows one to classify the diquark states in terms of $SU(2N_f) \otimes O(3)$ representations.

Let us do just this for the two flavour case $N_f = 2$ for both s- and p-wave states.

a) s-wave ground state $\sim \underline{10}_{10} \otimes \square_1$

The spin-flavour content of the $SU(4)$ diquark representation $\underline{10}$ can be determined by looking at how the $\underline{10}$ representation decomposes under

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the $SU(2)_{spin} \otimes SU(2)_{flavour}$ subgroup. One has

$$\square\square_{10} = \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}_1 \otimes \begin{smallmatrix}\square\\\square\end{smallmatrix}_1 + \square\square\square_3 \otimes \square\square\square_3 \right) \tag{1}$$

When coupling in the heavy quark one finally has the particle content

$$
\begin{array}{llll}
[q_1 q_2]: & 0^+ & \rightarrow \quad \frac{1}{2}^+ & \Lambda_Q \\[2mm]
\{q_1 q_2\}: & 1^+ & \nearrow\ \frac{1}{2}^+ \quad \Sigma_Q \\
 & & \searrow\ \frac{3}{2}^+ \quad \Sigma_Q^*
\end{array}
\tag{2}
$$

b) p-wave ($l_k = 0$, $l_K = 1$) $\sim \square\square_{10} \otimes \boxed{K}_3$: The $\underline{10}$ decomposes as before. The spin 0 and spin 1 pieces of the $\underline{10}$ couple with $l_K = 1$ to give $j^P = 1^-$ and $j^P = 0^-, 1^-, 2^-$, respectively. One has the particle content

$$
\begin{array}{llll}
[q_1 q_2]: & 1^- & \nearrow\ \left.\begin{smallmatrix}\frac{1}{2}^-\\[1mm]\frac{3}{2}^-\end{smallmatrix}\right\} & \{\Lambda_{QK1}^{**}\} \\[4mm]
\{q_1 q_2\}: & 0^- & \rightarrow\ \frac{1}{2}^- & \Sigma_{QK0}^{**} \\[3mm]
 & 1^- & \nearrow\ \left.\begin{smallmatrix}\frac{1}{2}^-\\[1mm]\frac{3}{2}^-\end{smallmatrix}\right\} & \{\Sigma_{QK1}^{**}\} \\[4mm]
 & 2^- & \nearrow\ \left.\begin{smallmatrix}\frac{3}{2}^-\\[1mm]\frac{5}{2}^-\end{smallmatrix}\right\} & \{\Sigma_{QK2}^{**}\} .
\end{array}
\tag{3}
$$

c) p-wave ($l_k = 1$, $l_K = 0$) $\sim \begin{smallmatrix}\square\\\square\end{smallmatrix}_6 \otimes \boxed{K}_3$: The diquark $\underline{6}$ of $SU(4)$ decomposes under the $SU(2)_{spin} \otimes SU(2)_{flavour}$ subgroup as

$$\begin{smallmatrix}\square\\\square\end{smallmatrix}_6 = \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}_1 \otimes \square\square\square_3 + \square\square\square_3 \otimes \begin{smallmatrix}\square\\\square\end{smallmatrix}_1 \right) . \tag{4}$$

After coupling with the k-orbital angular momentum $l_k = 1$ and the heavy quark spin $s = 1/2$, the particle content can then be determined to be

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$$[q_1 q_2]: \quad 0^- \; \rightarrow \; \tfrac{1}{2}^- \qquad \Lambda^{**}_{Qk0}$$

$$1^- \; \Big\langle \; \left.\begin{matrix} \tfrac{1}{2}^- \\[1em] \tfrac{3}{2}^- \end{matrix}\right\} \; \{\Lambda^{**}_{Qk1}\}$$

$$2^- \; \Big\langle \; \left.\begin{matrix} \tfrac{3}{2}^- \\[1em] \tfrac{5}{2}^- \end{matrix}\right\} \; \{\Lambda^{**}_{Qk2}\}$$

$$\{q_1 q_2\}: \quad 1^- \; \Big\langle \; \left.\begin{matrix} \tfrac{1}{2}^- \\[1em] \tfrac{3}{2}^- \end{matrix}\right\} \; \{\Sigma^{**}_{Qk1}\} \tag{5}$$

One thus has altogether seven Λ-type p-wave states and seven Σ-type p-wave states. The analysis can easily be extended to the case $SU(6) \otimes 0(3)$ bringing in the strangeness quark in addition.

Let us mention that, in the charm sector, the states $\Lambda_c(2285)$ and $\Sigma_c(2453)$ are well established while there is first evidence for the $\Sigma_c^*(2510)$ state. Recently two excited states $\Lambda_c^{**}(2593)$ and $\Lambda_c^{**}(2625)$ have been seen[9] which very likely correspond to the two p-wave states making up the $\{\Lambda_{cK1}^{**}\}$ Heavy Quark Symmetry doublet. The charm-strangeness states $\Xi_c(2470)$ and $\Omega_c(2720)$ have been seen and first evidence was presented for the $\tfrac{1}{2}^+$ $\Xi_c'(2570)$ state with the flavour configuration $c\{sq\}$. In the bottom sector, the $\Lambda_b(5640)$ has made its way into the Particle Data Booklet listing while some indirect evidence has been presented for the $\Xi_b(5800)$[10].

3. Heavy Baryon Wave Functions

We start by defining the Bethe-Salpeter amplitude (wave function) of a heavy baryon as

$$B_{\alpha\beta\gamma}(x_1, x_2, x_3) = \langle 0|T\psi_{q_1\alpha}(x_1)\psi_{q_2\beta}(x_2)\psi_{Q\gamma}(x_3)|B,P\rangle, \tag{6}$$

where $|B,P\rangle$ represents a particular heavy baryon state with mass M and momentum P. ψ_Q represents the heavy quark field, while the ψ_q's represent the light quark fields. α, β and γ are Dirac indices.

In a recent paper [11] we showed, using the LSZ reduction theorem and interpolating fields, along with the HQET, that the momentum space Bethe-Salpeter amplitude for a heavy baryon (i.e. the Fourier transform of (6)), can be written as

$$B_{\alpha\beta\gamma}(p_1, p_2, p_3) = \chi_{\rho\delta\gamma}(p_1, p_2, p_3) A^{\rho\delta}_{\alpha\beta}(p_1, p_2, p_3) , \qquad (7)$$

where the χ's are projection operators satisfying the Bargmann-Wigner equations

$$(\not{v} - 1)_\gamma{}^{\gamma'} \chi_{\rho\delta\gamma'} = 0 \qquad (8)$$

on each label. Here v^μ is the four velocity of the heavy baryon. These project out particular spin and parity states from the orbital wave functions A. We showed in [7] how to construct these projection operators for arbitrary meson and baryon orbital resonances. These χ's are reduced in terms of representations of $\mathcal{L} \otimes O(3,1)$, where $\mathcal{L}$ and $O(3,1)$, for baryons, are generated by

$$S_{\mu\nu} = \frac{1}{2}\sigma_{\mu\nu} \otimes \mathbb{1} \otimes \mathbb{1} + \mathbb{1} \otimes \frac{1}{2}\sigma_{\mu\nu} \otimes \mathbb{1} + \mathbb{1} \otimes \mathbb{1} \otimes \frac{1}{2}\sigma_{\mu\nu} \qquad (9)$$

and

$$L_{\mu\nu} = L^{(k)}_{\mu\nu} + L^{(K)}_{\mu\nu} , \qquad (10)$$

respectively where

$$L^{(k)}_{\mu\nu} = i(k_\mu \frac{\partial}{\partial k^\nu} - k_\nu \frac{\partial}{\partial k^\mu}) \qquad (11)$$

and similarly for $L^K_{\mu\nu}$. $S_{\mu\nu}$ acts on the Dirac labels. $L^{(k)}_{\mu\nu}$ is the angular momentum operator for the relative orbital angular momentum of the light quark pair, while $L^{(K)}_{\mu\nu}$ is the angular momentum operator for the relative orbital angular momentum between the centre of mass (c.m.) of the light quark pair and the heavy quark. Going to the rest frame of the baryon we reduce $O(3,1)$ to the rotation group $O(3)$. As we see there are two contributions to the orbital angular momentum. To find particular orbital angular momenta we look for appropriate eigenstates of $\vec{L}^2$. On the other hand, the group $\mathcal{L}$ is reduced to $SU(2)$ by use of the Bargmann-Wigner equations on each of the Dirac labels and by specifying the symmetry of these indices. Thus we end up with irreducible representations of $SU(2)_{spin} \otimes O(3)_{orbital}$. In the basis that we have chosen for the angular momenta, our procedure amounts to first adding the spin and orbital angular momenta of

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the light quarks to obtain the light diquark spin j and then adding in the Heavy Quark spin to obtain a pair of degenerate baryons with $J = j + 1/2$ and $J = j - 1/2$. The case $j = 0$ is special. In this case there is only one heavy baryon state with $J = 1/2$, as, for example, for the Λ-type baryon ground state Λ_Q.

The s-wave projection operators are now disposed of quite easily. The $\chi_{\alpha\beta\gamma}$ for s-waves can only be functions of $k_\perp^2$ or $K_\perp^2$. There are only two possibilites, either antisymmetric or symmetric in the $\alpha\beta$ indices,

$$\chi_{[\alpha\beta]\gamma}^\Lambda = \chi_{\alpha\beta}^0(v)\Psi_\gamma \tag{12}$$

or

$$\chi_{\{\alpha\beta\}\gamma}^\Sigma = \chi_{\alpha\beta}^{1,\mu}(v)\Psi_{\mu,\gamma}\,, \tag{13}$$

where $\chi_{\alpha\beta}^0(v) = \frac{1}{2\sqrt{2}}[(\slashed{v} + 1)\gamma_5 C]_{\alpha\beta}$ and $\chi_{\alpha\beta}^{1,\mu}(v) = \frac{1}{2\sqrt{2}}[(\slashed{v} + 1)\gamma_\perp^\mu C]_{\alpha\beta}$ with $\gamma_\perp^\mu = \gamma^\mu - \slashed{v}v^\mu$. Here C is the charge conjugation matrix. The normalisation is such that $(\bar{\chi}^0)^{\alpha\beta}\chi_{\alpha\beta}^0 = 1$ and $(\bar{\chi}^{1,\mu})^{\alpha\beta}\chi_{\alpha\beta}^{1,\nu} = -g_\perp^{\mu\nu}$, where $g_\perp^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$. The Bargmann-Wigner equations (8) ensure that

$$v^\mu\Psi_\mu = 0 \tag{14}$$

and

$$(\slashed{v} - 1)\Psi = (\slashed{v} - 1)\Psi_\mu = 0\,. \tag{15}$$

The superscripts Λ and Σ, in eqs. (12) and (13), indicate that these correspond to the projection operators of the Λ-type and Σ-type baryons respectively. The meaning of $\chi_{\alpha\beta}^0$ and $\chi_{\alpha\beta}^{1,\mu}$ is obvious. χ^0 represents the antisymmetric spin zero state of the light diquark, whereas $\chi^{1,\mu}$ represents the symmetric spin one state of the light diquark. The Λ-type baryons are also antisymmetric in the light flavours ensuring overall symmetry in $flavour \otimes spin \otimes space$ for the light diquark. Similarly the Σ-type baryons are symmetric in the light flavours. Then using the Bargmann-Wigner equations (15) we find that for the Λ baryon the spinor Ψ is the usual Dirac spinor i.e.

$$\Psi_\gamma = u_\gamma \tag{16}$$

whereas for the Σ-type spinor we see that (15) is solved by

$$\Psi_{\mu,\gamma} = \left\{ \begin{array}{c} u_{\mu,\gamma} \\ \frac{1}{\sqrt{3}}(\gamma_{\perp\mu}\gamma_5 u)_\gamma \end{array} \right\}\,. \tag{17}$$

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This corresponds to the Heavy Quark Symmetry Σ baryon doublet, $\frac{1}{2}^+$ and $\frac{3}{2}^+$. $u_{\mu,\gamma}$ is the Rarita-Schwinger spinor.

Going back to eq. (7) we can now write the Λ_Q s-wave baryon wave function as

$$B_{\alpha\beta\gamma} = \phi_{\alpha\beta}(v, k, K)u_\gamma \,, \tag{18}$$

where $\phi_{\alpha\beta}(v, k, K) = \chi^0_{\rho\delta}(v)A^{\rho\delta}_{\alpha\beta}(v, k, K)$.

In the same manner the wave function for the Σ-type s-wave heavy quark symmetry doublet can be written as

$$B_{\alpha\beta\gamma} = \phi^\mu_{\alpha\beta}(v, k, K) \left\{ \begin{array}{c} u_{\mu,\gamma} \\ \frac{1}{\sqrt{3}}(\gamma_{\perp\mu}\gamma_5 u)_\gamma \end{array} \right\} \tag{19}$$

where

$$\phi^\mu_{\alpha\beta}(v, k, K) = \chi^{1,\mu}_{\delta\rho}(v)A^{\delta\rho}_{\alpha\beta}(v, k, K) \,. \tag{20}$$

For p-waves the eigenvalue equation to be solved is

$$\vec{L}^2\chi = 2\chi \,. \tag{21}$$

There are two possibilities here. Either $(l_k = 1, l_K = 0)$ or $(l_k = 0, l_K = 1)$. Thus the above equation is solved either by

$$\chi(k) = k_{\perp\lambda}\bar{\chi}^\lambda \,. \tag{22}$$

or

$$\chi(K) = K_{\perp\lambda}\bar{\chi}^\lambda \,. \tag{23}$$

Here

$$k^\lambda_\perp = k^\lambda - v \cdot kv^\lambda \tag{24}$$

such that

$$v \cdot k_\perp = 0 \,. \tag{25}$$

Thus restituting the Dirac labels, the p-wave baryon projection operators are of the form $\chi_{\alpha\beta\gamma} = k_{\perp\lambda}\bar{\chi}^\lambda_{\alpha\beta\gamma}$ or $\chi_{\alpha\beta\gamma} = K_{\perp\lambda}\bar{\chi}^\lambda_{\alpha\beta\gamma}$. Note also that although both k and K are of mixed symmetry (under the interchange of p_1, p_2, p_3), k is symmetric under $p_1 \leftrightarrow p_2$ whereas K is symmetric. We then construct $\bar{\chi}^\lambda_{\alpha\beta\gamma}$ using the basic building blocks $\chi^0_{\alpha\beta}$ and $\chi^{1,\lambda}_{\alpha\beta}$, alongwith Dirac and generalised Rarita-Schwinger spinors, and ensuring overall symmetry with respect to $flavour \otimes spin \otimes orbital$ for the diquarks.

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A similar procedure can be followed for any orbital resonance where the eigenvalue equation to be solved is $\vec{L}^2\chi = l(l+1)\chi$. This has been worked out in detail in Ref. [7]. See also Ref. [8]. We now write down, in general, the covariant spin wave functions $B_{\alpha\beta\gamma}$ of the Heavy Quark Symmetry baryon doublets with $J = j + 1/2$ and $J = j - 1/2$ corresponding to the diquark spin j. One has [7,12]

$$B_{\alpha\beta\gamma} = \phi_{\alpha\beta}^{\mu_1\cdots\mu_j}(v, k, K) \left\{ \begin{array}{c} N_j(\gamma_{\perp\mu_1}\gamma_5 u_{\mu_2\cdots\mu_j})_\gamma \\ N'_j u_{\mu_1\cdots\mu_j;\gamma} \end{array} \right\} \tag{26}$$

where we have explicitly written out the Dirac spinor indices α, β and γ. As before the spinor indices α and β refer to the light quark system and the index γ refers to the heavy quark. Here

$$\phi_{\alpha\beta}^{\mu_1\cdots\mu_j}(v, k, K) = \hat{\phi}_{\delta\rho}^{\mu_1\cdots\mu_j}(v, k, K) A_{\alpha\beta}^{\delta\rho}(v, k, K), \tag{27}$$

where $\hat{\phi}$ are listed in Table 1 for p-waves. The general $\hat{\phi}$'s can be found in Ref.[7]. For the s-wave heavy Λ it is clear that $\hat{\phi}_{\alpha\beta} = \chi_{\alpha\beta}^0$ and similarly for the s-wave heavy Σ we have $\hat{\phi}_{\alpha\beta}^\mu = \chi_{\alpha\beta}^{1,\mu}$.

Dropping the spinor indices one has

$$B = \phi^{\mu_1\cdots\mu_j}\Psi_{\mu_1\cdots\mu_j} \tag{28}$$

where the "superfield" heavy baryon wave function $\Psi_{\mu_1\cdots\mu_j}$ stands for the two spin wave functions $\{j - 1/2, j + 1/2\}$ as indicated in Eq.(26). The normalizations (N_j and N'_j)are fixed by the normalization condition

$$\bar{\Psi}^{\mu_1\cdots\mu_j}\Psi_{\mu_1\cdots\mu_j} = (-1)^{J-1/2}2M \tag{29}$$

which gives $N_0 = 0$, $N_1 = \sqrt{1/3}$, $N_2 = \sqrt{1/10}$ and $N'_0 = 0$, $N'_1 = N'_2 = 1$ for the s- and p-wave cases discussed in this paper. There is an implicit understanding that the set of tensor indices "$\mu_1\cdots\mu_j$" is always completely symmetrized, traceless with regard to any pair of indices and transverse to the line of flight (i.e. to v^μ) in every index. This is shown explicitly in Table 1 where the s- and p-wave heavy baryon wave functions are listed. For example, the notation $\{\mu_1^\perp\mu_2^\perp\}_0$ implies symmetrization, tracelessness and transversity of the two tensor indices $\mu_1\mu_2$ as specified above. This specification is already implicit in the definition of the Rarita-Schwinger spinors $u_{\mu_1\cdots\mu_j}$ and is therefore not written out explicitly in these cases.

Table 1. Spin wave functions (s.w.f.) of heavy Λ-type and Σ-type s- and p-wave heavy baryons.

	light side s.w.f. $\hat{\phi}^{\mu_1...\mu_j}$	j^P	heavy side s.w.f. $\Psi_{\mu_1...\mu_j}$	J^P
s-wave states ($l_k = 0, l_K = 0$)				
Λ_Q	$\hat{\chi}$	0^+	u	$\frac{1}{2}^+$
$\{\Sigma_Q\}$	$\hat{\chi}^{1\mu_1}$	1^+	$\frac{1}{\sqrt{3}}\gamma^\perp_{\mu_1}\gamma_5 u$	$\frac{1}{2}^+$
			u_{μ_1}	$\frac{3}{2}^+$
p-wave states ($l_k = 0, l_K = 1$)				
$\{\Lambda^{**}_{QK1}\}$	$\hat{\chi}^0 K^{\mu_1}_\perp$	1^-	$\frac{1}{\sqrt{3}}\gamma^\perp_{\mu_1}\gamma_5 u$	$\frac{1}{2}^-$
			u_{μ_1}	$\frac{3}{2}^-$
Σ^{**}_{QK0}	$\frac{1}{\sqrt{3}}\hat{\chi}^1 \cdot K_\perp$	0^-	u	$\frac{1}{2}^-$
$\{\Sigma^{**}_{QK1}\}$	$\frac{i}{\sqrt{2}}\varepsilon(\mu_1\hat{\chi}^1 K_\perp v)$	1^-	$\frac{1}{\sqrt{3}}\gamma^\perp_{\mu_1}\gamma_5 u$	$\frac{1}{2}^-$
			u_{μ_1}	$\frac{3}{2}^-$
$\{\Sigma^{**}_{QK2}\}$	$\frac{1}{2}\{\hat{\chi}^{1,\mu_1} K^{\mu_2}_\perp\}_0$	2^-	$\frac{1}{\sqrt{10}}\gamma_5\gamma^\perp_{\{\mu_1} u_{\mu_2\}_0}$	$\frac{3}{2}^-$
			$u_{\mu_1\mu_2}$	$\frac{5}{2}^-$
p-wave states ($l_k = 1, l_K = 0$)				
$\{\Sigma^{**}_{Qk1}\}$	$\hat{\chi}^0 k^{\mu_1}_\perp$	1^-	$\frac{1}{\sqrt{3}}\gamma^\perp_{\mu_1}\gamma_5 u$	$\frac{1}{2}^-$
			u_{μ_1}	$\frac{3}{2}^-$
Λ^{**}_{Qk0}	$\frac{1}{\sqrt{3}}\hat{\chi}^1 \cdot k_\perp$	0^-	u	$\frac{1}{2}^-$
$\{\Lambda^{**}_{Qk1}\}$	$\frac{i}{\sqrt{2}}\varepsilon(\mu_1\hat{\chi}^1 k_\perp v)$	1^-	$\frac{1}{\sqrt{3}}\gamma^\perp_{\mu_1}\gamma_5 u$	$\frac{1}{2}^-$
			u_{μ_1}	$\frac{3}{2}^-$
$\{\Lambda^{**}_{Qk2}\}$	$\frac{1}{2}\{\hat{\chi}^{1,\mu_1} k^{\mu_2}_\perp\}_0$	2^-	$\frac{1}{\sqrt{10}}\gamma_5\gamma^\perp_{\{\mu_1} u_{\mu_2\}_0}$	$\frac{3}{2}^-$
			$u_{\mu_1\mu_2}$	$\frac{5}{2}^-$

The meaning of $\phi^{\mu_1\cdots\mu_j}$ is clear. It stands for the spin wave function of the light diquark with spin j. One can loosely call the $\Psi_{\mu_1\cdots\mu_j}$ the heavy side spin wave function. As mentioned before, $\phi^{\mu_1\cdots\mu_j}$ need not be specified any further, i.e. we do not need to know $A^{\delta\rho}_{\alpha\beta}(v,k,K)$, if we only wish to derive the consequences of the Heavy Quark Symmetry. In general these functions $A^{\delta\rho}_{\alpha\beta}(v,k,K)$ are different for different values of j. However, in the constituent quark model with $SU(2N_f)\otimes O(3)$ symmetry, we have $\phi^{\mu_1\cdots\mu_j}\to\hat{\phi}^{\mu_1\cdots\mu_j}$ [12] i.e. $A^{\delta\rho}_{\alpha\beta}(v,k,K)=\delta^\delta_\alpha\delta^\rho_\beta$.

From Table 1 we see that, for the p-wave spin wave functions, we can further explicitly pull out the relative momentum factor to write

$$\hat{\phi}^{\mu_1\cdots\mu_j}(v,p)=\hat{\phi}^{\mu_1\cdots\mu_j;\lambda}(v)p_\lambda^\perp\,,\tag{30}$$

where $p_\lambda^\perp=k_\lambda^\perp$ or $K_\lambda^\perp$. Now $\hat{\phi}^{\mu_1\cdots\mu_j;\lambda}(v)$ is only a function of v. Although we are only dealing with diquark spins $j=0,1,2$ in this paper the generic notation turns out to be quite convenient even for these simple cases. For the p-wave light-side spin wave functions one has the normalisation conditions

$$\left(\bar{\hat{\phi}}^{\nu_1\cdots\nu_j;\lambda}\right)^{\alpha\beta}\left(\hat{\phi}^{\mu_1\cdots\mu_j;\lambda'}\right)_{\alpha\beta}g^\perp_{\lambda\lambda'}=G^{\nu_1\cdots\nu_j;\mu_1\cdots\mu_j}\tag{31}$$

where $G^{\nu_1\cdots\nu_j;\mu_1\cdots\mu_j}$ is a generalised transverse metric tensor[7]. Fro example, for $j=0,1,2$ one has

$$j=0\quad G=1\tag{32}$$

$$j=1\quad G^{\mu\nu}=-g^{\mu\nu}_\perp\tag{33}$$

$$j=2\quad G^{\nu_1\nu_2;\mu_1\mu_2}=\frac{1}{2}(g^{\nu_1\mu_1}_\perp g^{\nu_2\mu_2}_\perp+g^{\nu_1\mu_2}_\perp g^{\nu_2\mu_1}_\perp-\frac{2}{3}g^{\nu_1\nu_2}_\perp g^{\mu_1\mu_2}_\perp)\tag{34}$$

One sees that the generic form of the normalisation condition (31) also covers the case of the s-wave normalisation with a slight adaption of notation.

For the full $SU(2N_f)$ wave functions one also needs to avail of the flavour diquark wave functions. For example, in the two flavour case (u,d) one has the flavour wave functions

$$D(I=0;I_3=0)=\frac{1}{\sqrt{2}}(ud-du),\qquad D(I=1;I_3=+1)=uu,$$

$$D(I=1;I_3=0)=\frac{1}{\sqrt{2}}(ud+du),\qquad D(I=1;I_3=-1)=dd.$$

For the applications we have in mind the corresponding flavour-space contractions are so simple that they are not listed separately. Also the extension to the three-flavour case (u, d, s) is straightforward.

4. Coupling Structure of Current Transitions

The heavy-side and light-side transitions for $b \to c$ transitions occur completely independent of each other (they "factorize") except for the requirement that the heavy side and the light side have the same velocity in the initial and final state, respectively, which are also the velocities of the initial and final heavy baryons. The $b \to c$ current transition, induced by the flavour-spinor matrix Γ, is hard in general and accordingly there is a change of velocities $v_1 \to v_2$. The heavy-side transitions are completely specified whereas the light-side transitions $j_1^{p_1} \to j_2^{p_2}$ are described by a number of form factors or coupling factors which parametrize the light-side transitions.

Now it is easy to write down the generic expressions for the current transitions following from the Heavy Quark symmetry limit [7,12]. One has

$$\bar{\Psi}^2_{\nu_1 \cdots \nu_{j_2}} \Gamma \Psi^1_{\mu_1 \cdots \mu_{j_1}} \left(\sum_{i=1}^{N} f^i(\omega) t_i^{\nu_1 \cdots \nu_{j_2}; \mu_1 \cdots \mu_{j_1}} \right) \tag{35}$$

where the $\Psi_{\mu_1 \cdots \mu_j}$ are the heavy baryon spin wave functions introduced in Sec. 1. The tensors $t_i^{\nu_1 \cdots \nu_{j_2}; \mu_1 \cdots \mu_{j_1}}$ describe the light side transitions. Here ω is the velocity transfer variable defined as $\omega = v_1 \cdot v_2$.

Thus the counting of the number of reduced form factors $f^i(\omega)$, that describe the heavy baryon transitions, can readily be done by referring to the number N of independant diquark transition amplitudes $t_i^{\nu_1 \cdots \nu_{j_2}; \mu_1 \cdots \mu_{j_1}}$ [12]. Defining the normality n of a diquark state with quantum numbers j^P by $n = P(-1)^j$ one has to differentiate between the two cases where the product of normalities of the two diquark states is even or odd. One finds

$$\begin{aligned} n_1 \cdot n_2 &= 1 & N &= j_{\min} + 1 \\ n_1 \cdot n_2 &= -1 & N &= j_{\min} \end{aligned}$$

The generic expression Eq.(35), completely determines the Heavy Quark Symmetry structure of the current transitions. What remains to be done is to write down independent sets of covariant coupling tensors. One notes that

Table 2. Tensor structure of the diquark transitions $j_1^{P_1} \to j_2^{P_2}$. Sign of the product of normalities $n_1 \cdot n_2$ and the smaller value of j_1, j_2 determine the number N of independent transitions or Isgur-Wise functions as specified in Eq.(35)

diquark transition				covariant amplitude
$j_1^{P_1} \to$	$j_2^{P_2}$	$n_1 \cdot n_2$	N	$t^{\nu_1 \dots \nu_{j_2}; \mu_1 \dots \mu_{j_1}}$
$0^+ \to$	0^+	$+1$	1	$f^{(0)} \cdot 1$
$0^+ \to$	0^-	-1	0	-
	1^-	$+1$	1	$f_{1,2}^{(1)} \cdot v_{1\nu_1}$
	2^-	-1	0	-
$1^+ \to$	1^+	$+1$	2	$-g_1^{(0)} \cdot g_{\mu_1 \nu_1} + g_2^{(0)} \cdot v_{1\nu_1} v_{2\mu_1}$
$1^+ \to$	0^-	$+1$	1	$g_2^{(1)} \cdot v_{2\mu_1}$
	1^-	-1	1	$ig_{1,3}^{(1)} \cdot \varepsilon(\mu_1 \nu_1 v_1 v_2)$
	2^-	$+1$	2	$-g_4^{(1)} \cdot v_{1\nu_1} g_{\nu_2 \mu_1} + g_5^{(1)} \cdot v_{1\nu_1} v_{1\nu_2} v_{1\mu_1}$

the heavy baryon transition amplitudes are factorized into a known heavy-side transition and an unknown light-side transition. We might add that a factorization of similar nature also occurs in other areas of particle physics as e.g. in the case of lepton-hadron interactions where one has a current-current interaction $j_\mu^{lepton} J_{hadron}^\mu$, and where the lepton current is known and the hadron current is parametrized by a set of unknown form factors.

The current transition tensors $t_i^{\nu_1 \dots \nu_{j_2}; \mu_1 \dots \mu_{j_1}}$ in Eq.(35) have to be built from the vectors $v_1^{\nu_i}$ and $v_2^{\mu_i}$, the metric tensor $g_{\mu_i \mu_k}$ and, depending on parity, from the Levi-Civita object $\varepsilon(\mu_i \nu_k v_1 v_2) := \varepsilon_{\mu_i \nu_k \alpha \beta} v_1^\alpha v_2^\beta$. The relevant basic tensors are listed in Table 2.

The number N of independent form factors for a given diquark transition can easily be obtained with the help of Eq.(35) and is also listed in Table 2. The column "covariant amplitudes" contains the basic tensors and the invariant or reduced amplitudes that multiply them. The nomenclature is such that the quasi-elastic $\Lambda_b \to \Lambda_c$ reduced form factor (or Isgur-Wise function) is

denoted by $f^{(0)}(\omega)$ with the zero recoil normalization condition $f^{(0)}(1) = 1$. The transitions to the p-wave states $\Lambda_b \to \{\Lambda_{cK1}^{**}\}$ and $\Lambda_b \to \{\Lambda_{ck1}^{**}\}$ are described by the form factors $f_1^{(1)}(\omega)$ and $f_2^{(1)}(\omega)$, respectively. For the Σ-type transitions there are two form factors each for the $\Sigma_b \to \{\Sigma_c\}$ and $\Sigma_b \to \{\Sigma_{cK2}^{**}\}$ transitions involving the diquark tarnsitions $1^+ \to 1^+$ and $1^+ \to 2^-$, respectively. The quasi-elastic $\Sigma_b \to \Sigma_c$ form factor $g_1^{(0)}(\omega)$ is normalized to 1 at zero recoil. The form factors $g_1^{(1)}(\omega)$ and $g_3^{(1)}(\omega)$, finally, describe the $\Sigma_b \to \Sigma_{ck1}^{**}$ and $\Sigma_b \to \Sigma_{cK1}^{**}$ transitions.

As an application we shall write down the contributions of the baryonic s- and p-wave states to the sum rule of Bjørken[13]. The calculation is straightforward yet cumbersome [7]. It involves the use of the covariance structure listed in Table 2 as well as the heavy baryon wave functions listed in Table 1. In the case of the $\Lambda_b \to \Lambda_c,\ \Lambda_c^{**}$ transitions one finds

$$1 = |f^{(0)}(\omega)|^2_{L=0} + (\omega^2 - 1)\left\{|f_1^{(1)}|^2_{L=1} + |f_2^{(1)}(\omega)|^2_{L=1}\right\} + \ldots \qquad (36)$$

where the ellipsis stand for the contributions of higher radial and orbital excitations not considered here, and for continuum contributions. For the Σ-type transitions $\Sigma_b \to \Sigma_c, \Sigma_c^*, \Sigma_c^{**}$ one has

$$\begin{aligned}
1 &= \frac{1}{9}|(\omega + 2)g_1^{(0)}(\omega) - (\omega^2 - 1)g_2^{(0)}(\omega)|^2_{L=0} \\
&+ \frac{2}{9}(\omega - 1)^2|g_1^{(0)}(\omega) - (\omega + 1)g_2^{(0)}(\omega)|^2_{L=2} \\
&+ (\omega^2 - 1)\{\frac{2}{3}|g_1^{(1)}|^2_{L=1} + \frac{1}{3}|g_2^{(1)}|^2_{L=1} + \frac{4}{3}|g_3^{(1)}(\omega)|^2_{L=1} \\
&+ \frac{8}{45}|(\omega + \frac{3}{2})g_4^{(1)} - (\omega^2 - 1)g_5^{(1)}|^2_{L=1} \\
&+ \frac{12}{45}(\omega - 1)^2|g_4^{(1)}(\omega) - (\omega + 1)g_5^{(1)}|^2_{L=3}\}\ldots \qquad (37)
\end{aligned}$$

For the $\Sigma_b \to \{\Sigma_c, \Sigma_c^*\}$ and the $\Sigma_b \to \{\Sigma_{cK2}^{**}\}$ transitions we have used a diagonal basis in terms of the partial wave amplitudes as indicated in Eq.(37). The subscript L in equations (36) and (37) refer to the different partial wave contributions. We see that the contributions of the various Λ-type and Σ-type states exhibit the characteristic $p^{2L} \propto (\omega - 1)^L$ threshold powers of the diquark transitions.

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The presentation of the Bjørken sum rules completes our discussion of the Heavy Quark Symmetry structure of current-induced heavy baryon transitions. The set of reduced form factors constitute the maximal possible reduction of the full complexity of the form factor problem which can be achieved using just the Heavy Quark Symmetry. To obtain further information on the reduced form factors one necessarily has to study the internal dynamics of the light diquark system itself. This is a notoriously difficult problem since it involves the solution of quark-gluon dynamics in the non-perturbative regime. First attempts at solving this problem in the baryonic sector are being undertaken using lattice techniques, QCD sum rule methods, or, more conventionally, using some potential type quark model. Short of doing a full-fledged dynamical calculation one can work in the context of the constituent quark model and try to extract as much information on the diquark transitions using the $SU(2N_f) \otimes O(3)$ symmetry classification of the diquark states. This is the subject of the next section.

5. $SU(2N_f) \otimes O(3)$ **Structure of the Light-Side Transitions**

For the s-wave to s-wave transition the relevant matrix element for the light side transition is given by

$$\sum_{i=1}^{N} f^i(\omega) t_i^{\nu_1 \cdots \nu_{j_2}; \mu_1 \cdots \mu_{j_1}} = \left(\bar{\hat{\phi}}^{\nu_1 \cdots \nu_{j_2}}(v_2) \right)^{\alpha' \beta'} (\mathcal{O})_{\alpha' \beta'}^{\alpha \beta} \left(\hat{\phi}^{\mu_1 \cdots \mu_{j_1}}(v_1) \right)_{\alpha \beta}, \quad (38)$$

where the $\hat{\phi}$'s are either χ^0 or $\chi^{1,\mu}$, i.e. they either have zero indices or just one index, depending on whether we have a Λ_b to Λ_c transition or Σ_b to Σ_c transition. $\mathcal{O}$ is the operator given by the overlap integral

$$\mathcal{O}_{\alpha' \beta'}^{\alpha \beta} = \int d^4 k_2 d^4 K_2 d^4 k_1 d^4 K_1 \bar{A}_{\alpha' \beta'}^{\rho \sigma} \mathcal{T}_{\rho \sigma}^{\rho' \sigma'} A_{\rho' \sigma'}^{\alpha \beta}. \quad (39)$$

Here $\mathcal{T}_{\rho \sigma}^{\rho' \sigma'}(k_2, K_2; k_1, K_1)$ is the transition kernel. k_1, K_1 and k_2, K_2 are repectively the relative momenta k, K for the incoming and outgoing quarks. This overlap integral is common for both Λ and Σ type transitions but this information is not of much help. It can be seen, from general arguments of the Lorentz structure, that $\mathcal{O}$ in general consists of five independent contributions [4] $\mathbb{1} \otimes \mathbb{1}$, $\gamma_5 \otimes \gamma_5$, $\gamma_\mu \otimes \gamma^\mu$, $\gamma_\mu \gamma_5 \otimes \gamma^\mu \gamma_5$ and $\sigma_{\mu\nu} \otimes \sigma^{\mu\nu}$,[*] which we

_____

[*]Notice that there are no $\not{v}_1$ or $\not{v}_2$ terms as these reduce to $\mathbb{1}$ when acting on the projectors.

call the S, P, V, A and T couplings of the spins of the two spectator systems of diquarks. Here the S operator is a one-body operator whereas the others are two-body operators involving interactions between the light quarks. This set can be divided into the three antisymmetric independent combinations $2S - \frac{1}{2}T + A$, $V + A$ and $S - V - P$ contributing to the Λ transitions and the two symmetric combinations $2S + V - A - 2P$ and $3S + \frac{1}{2}T + 3P$ contributing to the Σ transitions [4]. In this way way we get no relation between the Λ and Σ transition form factors.

Similarly for the transitions from the ground state bottom baryon to the p-wave charm baryon states the relevant light side matrix element is given by

$$\left(\bar{\hat{\phi}}^{\nu_1...\nu_{j_2};\nu}(v_2)\right)^{\rho'\sigma'} (\mathcal{O}_\nu^p)_{\rho'\sigma'}^{\rho\sigma} \left(\hat{\phi}^{\mu_1...\mu_{j_1}}(v_1)\right)_{\rho\sigma}. \tag{40}$$

Here $\mathcal{O}_\nu^p$ is either $\mathcal{O}_\nu^K$ or $\mathcal{O}_\nu^k$ with

$$(\mathcal{O}_\nu^p)_{\alpha'\beta'}^{\alpha\beta} = \int d^4k_2 d^4K_2 d^4k_1 d^4K_1 \; p_\nu^\perp \bar{A}_{\alpha'\beta'}^{\rho\sigma} \mathcal{T}_{\rho\sigma}^{\rho'\sigma'} A_{\rho'\sigma'}^{\alpha\beta}, \tag{41}$$

where $p_\perp$ is transverse to the outgoing velocity v_2. Here again in general $\mathcal{O}_\nu^p$ can consist of the one-body operators $v_{1\nu}^\perp \mathbb{1} \otimes \mathbb{1}$ and $(\mathbb{1} \otimes \gamma_\nu^\perp \pm \gamma_\nu^\perp \otimes \mathbb{1})$ plus a large number of two-body operators $v_{1\nu}^\perp \gamma^\mu \otimes \gamma_\mu$, etc. This means that we would not get any more relations beyond those already given in Table 2.

However, recently interest in the constituent quark model has been rekindled by the discovery (or rediscovery) that two-body spin-spin interactions between quarks are nonleading in $1/N_C$, at least in the baryon sector [14,15]. Thus, to leading order in $1/N_C$, light quarks behave as if they were heavy and a classification of a light quark system in terms of $SU(2N_f) \otimes O(3)$ symmetry multiplets makes sense. In this case transitions between light quark systems are parametrized only in terms of the set of one-body operators whose matrix elements are then evaluated between the $SU(2N_f) \otimes O(3)$ multiplets.

For the diquark transitions discussed in this paper, the relevant one-body operators are then given by

s-wave to s-wave:

$$O = A(\omega) \mathbb{1} \otimes \mathbb{1} \tag{42}$$

s-wave to p-wave: Here we distinguish transitions from the ground state to the two types of p-wave excitations $(l_K = 1; l_k = 0)$ and $(l_K = 0; l_k = 1)$, denoted by the superscripts K and k respectively,

$$\mathcal{O}_\nu^K = A^K(\omega)\, v_{1\nu}^\perp \mathbb{1} \otimes \mathbb{1} + B_K(\omega)\,(\mathbb{1}\otimes \gamma_\nu^\perp + \gamma_\nu^\perp \otimes \mathbb{1})$$

$$\mathcal{O}_\nu^k = A^k(\omega)\, v_{1\nu}^\perp \mathbb{1} \otimes \mathbb{1} + B_k(\omega)\,(\mathbb{1}\otimes \gamma_\nu^\perp - \gamma_\nu^\perp \otimes \mathbb{1}) \tag{43}$$

The reduced form factors in Eqs.(42) and (43) depend on the velocity transfer variable ω and are unknown functions expect for the normalization condition $A(1) = 1$ in Eq.(42). The operators $\mathbb{1}\otimes\mathbb{1}$ and $v_{1\nu}^\perp \mathbb{1}\otimes\mathbb{1}$ do not couple angular and spin degrees of freedom. These were the operators used some thirty years ago when the consequences of the collinear symmetry $SU(6)_W$ or $\tilde{U}(12)$ were worked out. In the following we shall therefore refer to the A-type operators as the collinear operators. The operators $(\mathbb{1}\otimes \gamma_\nu^\perp \pm \gamma_\nu^\perp \otimes \mathbb{1})$ on the other hand, introduce spin-orbit coupling interactions and will therefore be called spin-orbit operators.

The matrix elements, Eq.(38) and Eq.(40), of the operators Eq.(42) and (43) can be readily evaluated using the light-side spin wave functions in Table 1. The three ground state to ground state form factors $f^{(0)}(\omega)$, $g_1^{(0)}(\omega)$ and $g_2^{(0)}(\omega)$ can now be expressed by the single form factor $A(\omega)$ as

$$
\begin{aligned}
f^{(0)}(\omega) &= \frac{\omega+1}{2} A(\omega) \\
g_1^{(0)}(\omega) &= \frac{\omega+1}{2} A(\omega) \\
g_2^{(0)}(\omega) &= \frac{1}{2} A(\omega)
\end{aligned}
\tag{44}
$$

This result has been derived before in Ref.[4] and agrees with the large-N_C predictions, reported in Ref. [11], where a universal form factor $A(\omega)$ (with normalization $A(1) = 1$) determines the semileptonic $\Lambda_b \to \Lambda_c$ and $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$ transitions. Looking at the Bjørken sum rule, Eq.(37), it is clear, in this case, that the Σ-type transitions are predicted to result from pure $L = 0$ diquark transitions. This is a testable prediction in as much as the population of the helicity states in the decay baryon is fixed resulting in a characteristic angular decay pattern of the subsequent decays. More difficult is a test of the relation between the Λ-type and the Σ-type form factors. In the test one would have to compare $\Lambda_b \to \Lambda_c$ and $\Omega_b \to \{\Omega_c,\ \Omega_c^*\}$ transitions (where there are additional $SU(3)$ breaking effects) since these are the transitions that are

experimentally accessible. We mention that the $\Sigma_b \rightarrow \{\Sigma_c, \Sigma_c^*\}$ branching fraction is expected to be too small to be measurable.

For the s to p transitions the evaluation of the matrix element (40) is straightforward and one obtains
$(l_K = 1; \; l_k = 0)$

$$
\begin{aligned}
f_1^{(1)}(\omega) &= \frac{\omega + 1}{2} A^K(\omega) + B^K(\omega) \\
g_2^{(1)}(\omega) &= \frac{1}{\sqrt{3}} \left(\frac{\omega + 1}{2} A^K(\omega) + 3 B^K(\omega) \right) \\
g_3^{(1)}(\omega) &= -\frac{1}{\sqrt{2}} \left(\frac{\omega + 1}{2} A^K(\omega) + B^K(\omega) \right) \\
g_4^{(1)}(\omega) &= \frac{\omega + 1}{2} A^K(\omega) \\
g_5^{(1)}(\omega) &= \frac{1}{2} A^K(\omega)
\end{aligned}
\tag{45}
$$

$(l_K = 0; \; l_k = 1)$

$$
\begin{aligned}
f_2^{(1)}(\omega) &= \sqrt{2} B^k(\omega) \\
g_1^{(1)}(\omega) &= -B^k(\omega)
\end{aligned}
\tag{46}
$$

These relations include but go beyond the Heavy Quark Symmetry relations given in Table 2. We see that the five form factors $f_1^{(1)}, g_{2-5}^{(1)}$ are now expressed in terms of just two unknown functions whereas the two form factors $f_2^{(1)}$ and $g_1^{(1)}$ are reduced to just one unknown function B^k.

From the equation (37) we see that the "partial wave" amplitudes for the case $1^+ \rightarrow 2^-$ are related to the form factors through

$$
\begin{aligned}
A_{L=1} &\propto (\omega + \frac{3}{2}) g_4^{(1)}) - (\omega^2 - 1) g_5^{(1)} \\
A_{L=3} &\propto g_4^{(1)} - (\omega + 1) g_5^{(1)} .
\end{aligned}
\tag{47}
$$

Putting in the expressions from Eqs.(45) we see that the $A^K(\omega)$ contribution is purely $L = 1$. One therefore predicts that only the lower partial wave $L = 1$ is populated in the $\Sigma_b \rightarrow \{\Sigma_{cK2}^{**}\}$ transition. As before this has implications for the population of density matrix elements of the daughter baryons that can be experimentally checked. In the collinear limit, where $B^k = 0$ and

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$B^K = 0$, the transitions to the p-wave k-multiplet are completely suppressed and there are simple relations for the transitions into the K-muliplet. It would be interesting to experimentally determine the relative strengths of the collinear and spin-orbit. From naive expectations one would expect the spin-orbit contributions to be suppressed relative to the leading collinear terms. Let us finally remark that the transitions to the k- and K-multiplets become related when one has a complete factorization of the light quark wave functions ("independent quark motion")[4].

6. Concluding Remarks

We have studied the consequences of Heavy Quark Symmetry for current transitions between heavy baryons. For these transitions we discussed how the most general Heavy Quark Symmetry structure can be further simplified by invoking a constituent quark model $SU(2N_f) \otimes O(3)$ symmetry for the light side transition. The key ingredient is that the transitions factorize into a known heavy-side transition and an unknown light-side transition when $m_Q \to \infty$. The light-side and heavy-side transitions are linked through angular coupling factors which are calculable. This factorization breaks down when higher order $1/m_Q$ effects are taken into account. In this case the light-side and heavy-side transitions become entangled in a very nontrivial way. Depending on one's own attitude such additional complications may be considered a curse or a blessing. From this point of view the charm sector ($\Lambda_{QCD}/m_c \simeq (20-30)\%$) is best suited for studying O($1/m_Q$) effects whereas the bottom sector ($\Lambda_{QCD}/m_b \simeq (6-10)\%$) would be best suited to see how Heavy Quark Symmetry is at work.

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